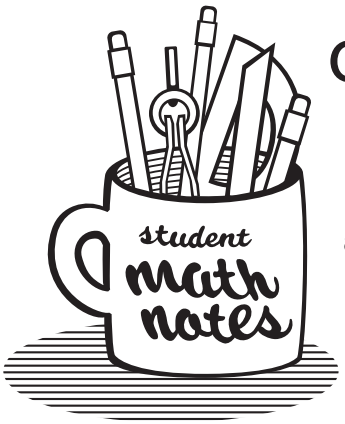
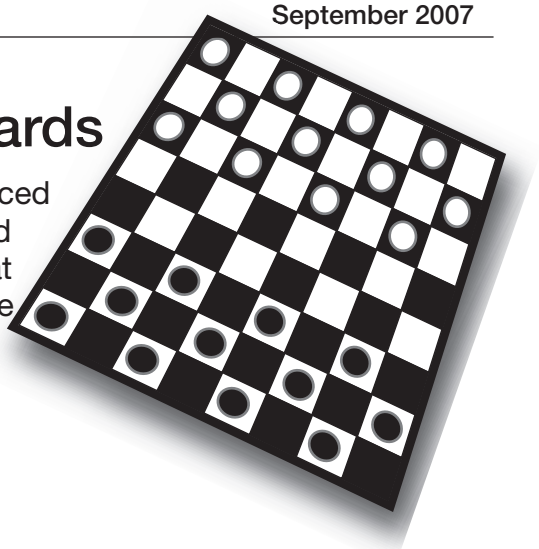


Checking Out Checkerboards



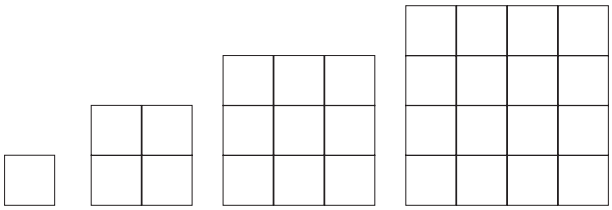
The game of checkers, or draughts (pronounced "drafts"), as it is known in Great Britain and Scotland, has ancient roots. It is thought that archeologists excavating in Iraq unearthed the earliest form of checkers as a game. Today's checkerboard uses red-and-black pieces played on an 8 × 8 board.

Think about how many squares are on a standard 8 × 8 checkerboard.

Many people count just the small squares and answer 64. Some remember that the entire board is one large square and answer 65. In fact, there are many more. Where are all these other squares? How do we count them?

One problem-solving strategy is to solve a simpler problem and look for patterns. Let's begin by imagining smaller boards and counting the squares on them.

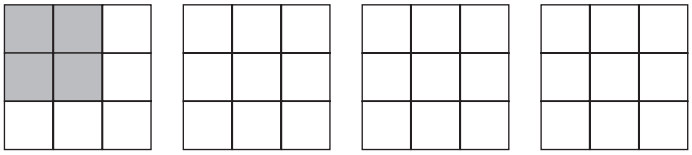
The figures below show a set of smaller boards composed of unit squares. Notice the boards are named according to the lengths of their sides: a 1 × 1 board measures 1 square along each side, a 2 × 2 board measures 2 squares along each side, and so forth.



The first figure has only one square. How many squares are in the second figure? Remember to count the entire 2 × 2 figure as a square. The second figure has five squares in all: four 1 × 1 squares and one 2 × 2 square.

Squares on a 3 × 3 Board

1. Find the number of squares on a 3 × 3 board (next coluumn). How many different-sized squares are there? If you have trouble counting the 2 × 2 squares, shade the 2 × 2 squares on the boards.



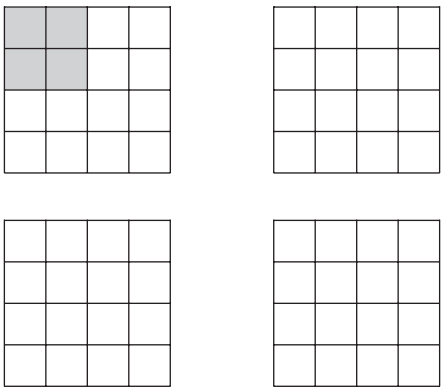
Use the table below to record the number of each of the different-sized squares.

Squares on a 3 × 3 Board	
Size of Square	**Number of Squares**
1 × 1	
2 × 2	
3 × 3	

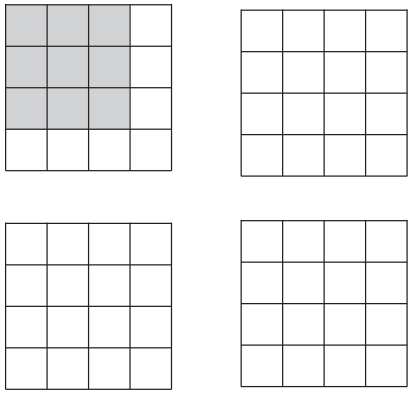
Apply a Similar Strategy to Other Boards

2. How many squares are on a 4 × 4 board? Let's explore!

   a. Continue to shade the 2 × 2 squares on the boards as shown below. You might want to use grid paper to draw boxes around 4 × 4 grids.



b. How many 3 × 3 squares are there? You can use the strategy above to find the number of 3 × 3 squares.



c. Complete the table to determine how many squares are on a 4 × 4 board.

Number of Squares on Various Boards				
Size of Square	1 × 1 Board	2 × 2 Board	3 × 3 Board	4 × 4 Board
1 × 1	1	4	9	
2 × 2		1		
3 × 3				
4 × 4				
Totals				

To determine how many squares are on a 5 × 5 board, you could draw a picture and begin to count, but you may find this process to be time consuming. When you reach a point where a strategy is inefficient, it is sometimes helpful to organize your data and look for patterns.

3. Look at the table you created in problem 2. What pattern(s) do you see?

4. Certain numbers repeat along the diagonals of the table. What do you notice about each of these numbers? (Hint: Remember you are counting squares.)

5. The table below has been extended to determine the number of squares on a 5 × 5 board. How many squares are on a 5 × 5 board?

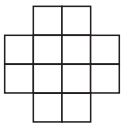
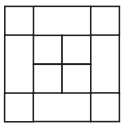
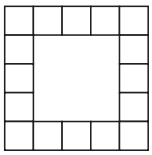
Squares on Various Boards					
Size of Square	1 × 1	2 × 2	3 × 3	4 × 4	5 × 5
1 × 1	1	4	9	16	25
2 × 2		1			
3 × 3					
4 × 4					
5 × 5					
Totals					

6. Use this pattern to solve the original checkerboard problem. A standard checkerboard is 8 × 8. What is the total number of squares? Is this a reasonable answer? (Hint: You may find it helpful to use the table below.)

Squares on Various Boards								
Size of Square	1×1	2×2	3×3	4×4	5×5	6×6	7×7	8×8
1 × 1	1	4	9	16	25			
2 × 2		1						
3 × 3								
4 × 4								
5 × 5								
6 × 6								
7 × 7								
8 × 8								
Totals								

7. How could you find how many squares are on any size square board? Write a formula or describe a procedure for finding the answer.

8. The following game boards were created by groups of students as a class project on designing strategy games. What is the total number of squares on each game board?

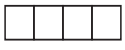
a)  *b*)  *c*) 

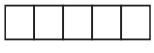
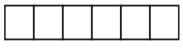
What strategies did you use to find the total number of squares on each game board?

Finding Rectangles

9. The figures below were made using unit squares. What is the total number of *rectangles* in each figure? (Remember, a square is a type of rectangle).

a)  *b*) 

c)  *d*) 

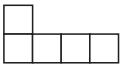
e)  *f*) 

What patterns do you see?

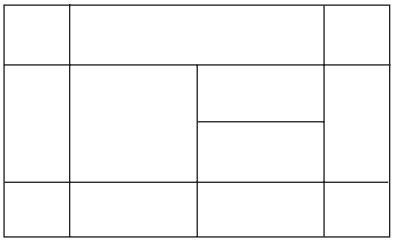
10. *a*. Find the number of rectangles in a 10 × 1 figure.

b. Write an expresion for finding the total number of rectangles in a 1 × *n* figure

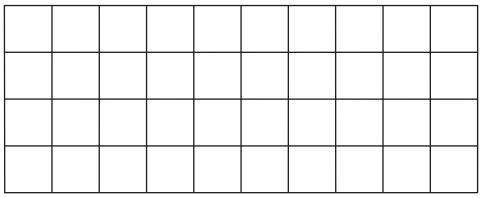
11. Determine the total number of rectangles in each figure.

a)  *b*)  *c*)  *d*) 

12. Floors can be tiled using many different patterns. The following board represents a floor tile pattern. How many total squares can you find?

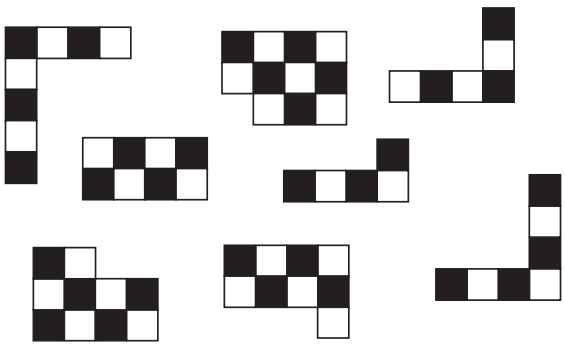


13. Use the grid provided to create your own tile pattern that has exactly 5 squares.

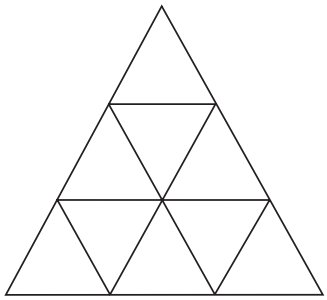


Can You . . .

- find the 1296 rectangles in a standard 8 × 8 checkerboard?
- put the 8 pieces together to form a standard 8 × 8 checkerboard?



- determine if this figure has more triangles, parallelograms, or trapezoids?



Did You Know That . . .

- carbon dating makes it appear that this game was played around 3000 BC? However, the game used a slightly different board and a different number of pieces, and the exact rules are uncertain.
- first references to the checkers game are found as early as 1600 BC in Egyptian paintings and inscriptions during the time of the Pharaohs?
- the first world championship in checkers was awarded in 1847?
- checkers has been played on boards of different sizes around the world (8 × 8, 10 × 10, and 12 × 12)?
- the game of international checkers is played on a 10 × 10 board?

Mathematical Content

Patterns, organizing information using a table, visualization

Resources

Bourassa, Ed. "Middle Level Mathematics Encouraging an Algebraic Mindset." mathcentral.uregina.ca/RR/database/RR.09.97/bourassa1.html.

Gardner, Martin, ed. *Mathematics Puzzles of Sam Lloyd*. New York: Dover Publications, 1959.

PlayJava.com. "Checkers, Game Basics, History and Rules." 1999, 2000. www.playjava.com/checkers_game_basics.html.

Rayment, W. J. "History of Checkers or Draughts." 2004–2007. www.indepthinfo.com/checkers/history.shtml.

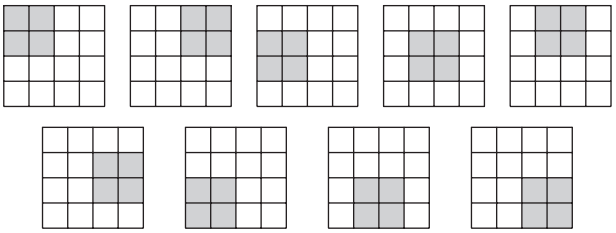
www.acfcheckers.com/origin.html

Checking Out Checkerboards—*continued*

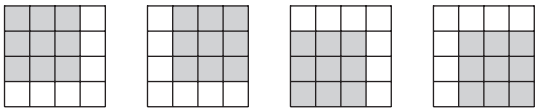
Answers

1. 9, 4, 1, Total: 14

2a). 9; 2 × 2 squares on a 4 × 4 board



b) 4; 3 × 3 squares on a 4 × 4 board



c)

Number of Squares on Various Boards				
Size of Square	1 × 1 Board	2 × 2 Board	3 × 3 Board	4 × 4 Board
1 × 1	1	4	9	16
2 × 2		1	4	9
3 × 3			1	4
4 × 4				1
Totals	1	5	14	30

3. Each number of squares increases by the next square number. Each entry in the table is a perfect square.

4. Answers may vary. Sample: They increase by consecutive square numbers. They are perfect squares.

5.

Squares on Various Boards					
Size of Square	1 × 1	2 × 2	3 × 3	4 × 4	5 × 5
1 × 1	1	4	9	16	25
2 × 2		1	4	9	16
3 × 3			1	4	9
4 × 4				1	4
5 × 5					1
Totals	1	5	14	30	55

6. Answers may vary.

Squares on Various Boards								
Size of Square	1×1	2×2	3×3	4×4	5×5	6×6	7×7	8×8
1 × 1	1	4	9	16	25	36	49	64
2 × 2		1	4	9	16	25	36	49
3 × 3			1	4	9	16	25	36
4 × 4				1	4	9	16	25
5 × 5					1	4	9	16
6 × 6						1	4	9
7 × 7							1	4
8 × 8								1
Totals	1	5	14	30	55	91	140	204

7. Answers may vary.
$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + \ldots + n^2$, or
total number of squares is $n^2 + (n-1)^2 + \ldots + 1^2$
Add all the perfect squares up to the size of the board (an $n \times n$ board would include the sum of all perfect squares up to n^2).

8. *a.* 17 *b.*18 *c.* 22

9. *a.* 1
b. 1 + 2 = 3
c. 1 + 2 + 3 = 6
d. 1 + 2 + 3 + 4 = 10
e. 1 + 2 + 3 + 4 + 5 = 15
f. 1 + 2 + 3 + 4 + 5 + 6 = 21

The number of rectangles is the sum of the counting numbers up to the number of unit squares in the figure, or $1 + 2 + 3 + \ldots + (n\text{-}1) + n$ for a figure that is $1 \times n$.

10. a) 55; 1 + 2 + 3 + 4 10

b) $\frac{n(n+1)}{2}$

11. *a.* 12 *b.* 11 *c.* 9 *d.* 18

12. 10 squares total: four 1 × 1, three 2 × 2, two 3 × 3, one 4 × 4

13. Answers may vary.

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